

# Ultrashort Pulse Interaction with Intersubband Transitions of Semiconductor Quantum Wells

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**SCOPE OF THIS WORK:** We study the problem of coherent ultrashort pulse propagation in a two-subband system in a symmetric semiconductor quantum well structure [1], performing calculations beyond the rotating wave approximation (RWA) and the slowly varying envelope approximation (SVEA) [2,3] and taking into account the effects of electron-electron interactions [3-5]. The interaction of the quantum well structure with the electromagnetic field is studied with modified, nonlinear, Bloch equations [6]. These equations are combined with the full-wave Maxwell equations for the study of pulse propagation, so the coupled Maxwell-nonlinear Bloch equations are solved computationally beyond the RWA and SVEA. We present results for the pulse propagation and the population inversion dynamics in the quantum well structure for electromagnetic pulses for a  $2\pi$  pulse at the entrance of the medium and for different electron sheet densities.

## EFFECTIVE NONLINEAR OPTICAL BLOCH EQUATIONS FOR INTERSUBBAND TRANSITION DYNAMICS IN A SEMICONDUCTOR QUANTUM WELL AND MAXWELL-WAVE EQUATIONS

$$\partial S_1 / \partial t = (\omega_{10} - \gamma S_3) S_2 - S_1 / T_2$$

$$\partial S_2 / \partial t = -(\omega_{10} - \gamma S_3) S_1 + 2(\mu E_x / \hbar - \beta S_1) S_3 - S_2 / T_2$$

$$\partial S_3 / \partial t = -2(\mu E_x / \hbar - \beta S_1) S_2 - (S_3 + 1) / T_1$$

$$\frac{\partial E_x}{\partial t} = -\frac{1}{\varepsilon_r} \frac{\partial H_y}{\partial z} - \frac{1}{\varepsilon_r} \frac{\partial P_x}{\partial t}$$

$$\frac{\partial H_y}{\partial t} = -\frac{1}{\mu_r} \frac{\partial E_x}{\partial z}$$

•  $\mu$  is the electric dipole matrix element between the two subbands.  $E_x(z,t)$  is the total electric field applied to the quantum well structure.

•  $\omega_{10}$ ,  $\beta$ ,  $\gamma$  are parameters defined by means of the envelope functions of the ground and excited states in the quantum well system and depend on electron sheet density  $N$ .  $N_v$  is the electron volume density.

• The relaxation processes are described by the population delay time  $T_1$  and the dephasing time  $T_2$ .

•  $\varepsilon_r$  is the relative dielectric constant and  $\mu_r$  is the magnetic permeability of the medium.

$$\omega_{10} = \frac{E_{10}}{\hbar} + \frac{\pi e^2 N}{2\hbar \varepsilon_r} (L_{1111} - L_{0000})$$

$$\gamma = \frac{\pi e^2 N}{2\hbar \varepsilon_r} (2L_{1001} - L_{1111} - L_{0000}), \quad \beta = \frac{\pi e^2 N}{\hbar \varepsilon_r} L_{1100}$$

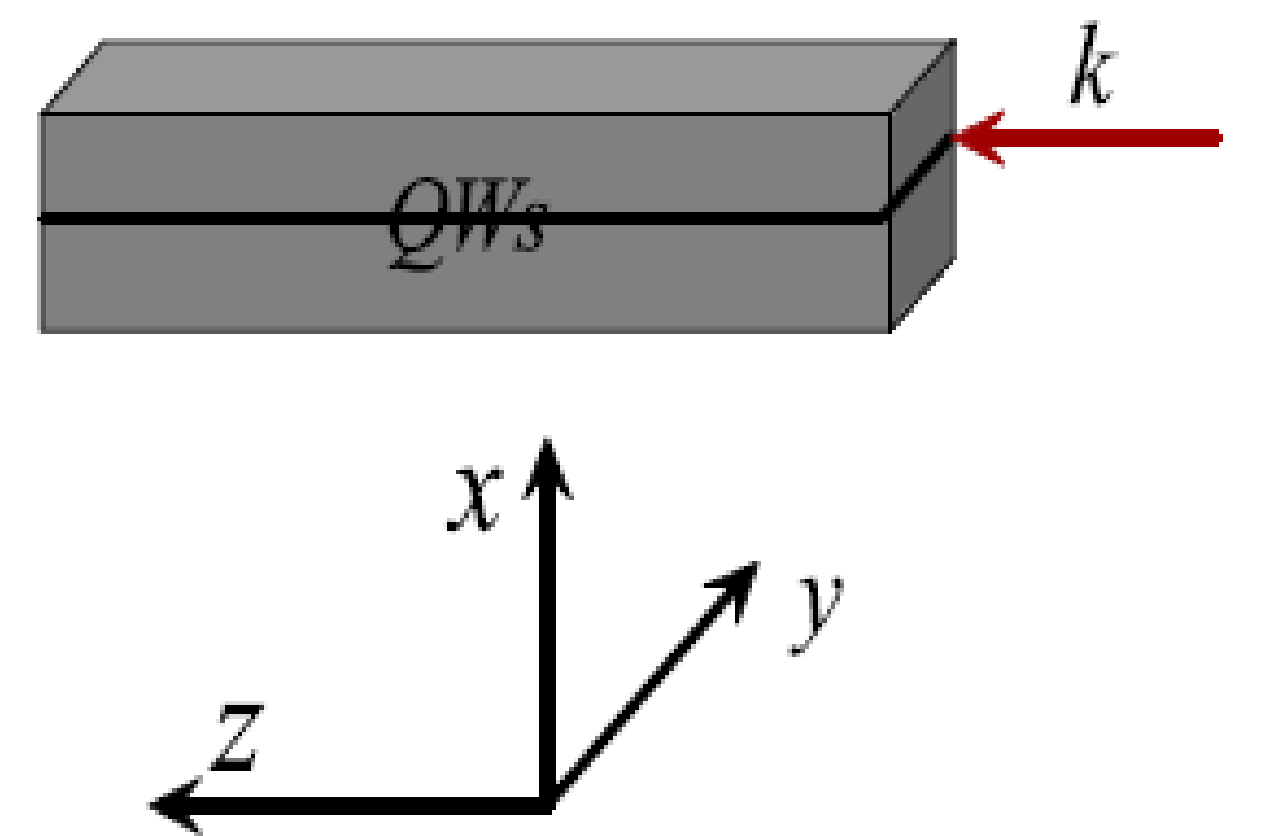
$$L_{ijkl} = \iint dx dx' \xi_i(x) \xi_j(x') |x - x'| \xi_k(x') \xi_l(x)$$

$$E_x(z=0,t) = E_0 \operatorname{sech}[1.76(t-t_0)/t_p] \cos[\omega(t-t_0)]$$

**QUANTUM WELL STRUCTURE UNDER STUDY:** We consider a GaAs/AlGaAs double quantum well [3-6]. The structure consists of two GaAs symmetric square wells of width 5.5 nm and height 219 meV. The wells are separated by a AlGaAs barrier of width 1.1 nm. For this system we take  $T_1 = 100$  ps and  $T_2 = 10$  ps. We study the interaction of the quantum well structure with an initially hyperbolic secant electromagnetic pulse and calculate the dynamics of the population inversion for various pulse durations and electron sheet densities. For the structure under study we present results in the region  $N = 10^9 - 5 \times 10^{11} \text{ cm}^{-2}$ , such that the system can be initially taken in the ground subband. We consider a structure with 40 quantum wells, each one equally separated by the other by 20 nm in a AlGaAs substrate, along the  $z$  axis.

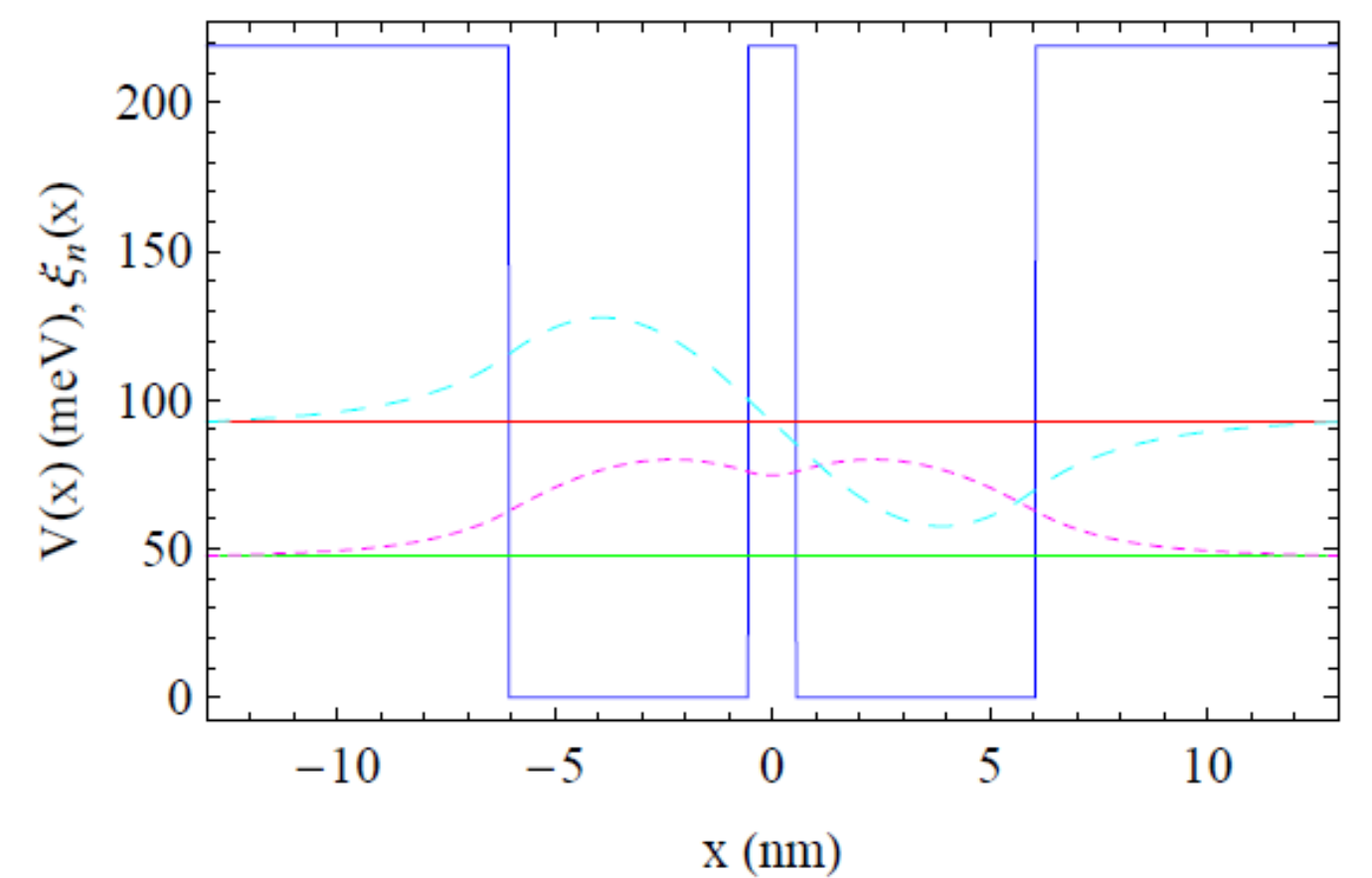
**COMPUTATIONAL PROCEDURE:** We first solve numerically the time-independent Schrödinger equation, in the effective mass approximation, using the shooting method [7] and calculate the parameters of the quantum well that enter into Bloch equations. Then, we solve the coupled Maxwell-nonlinear Bloch equations computationally, without applying the SVEA and the RWA, using a combination of the finite-difference time-domain method for the Maxwell equations and a predictor-corrector scheme for the nonlinear Bloch equations [2,3]. In the propagation homogenization of the structure is considered, similar to what was done in previous studies [2,3].

**Right:** Propagation geometry. Pulses are injected with the wave vectors  $k$  parallel to the plane of the quantum wells ( $z$  axis), and  $x$  direction denotes the growth axis.

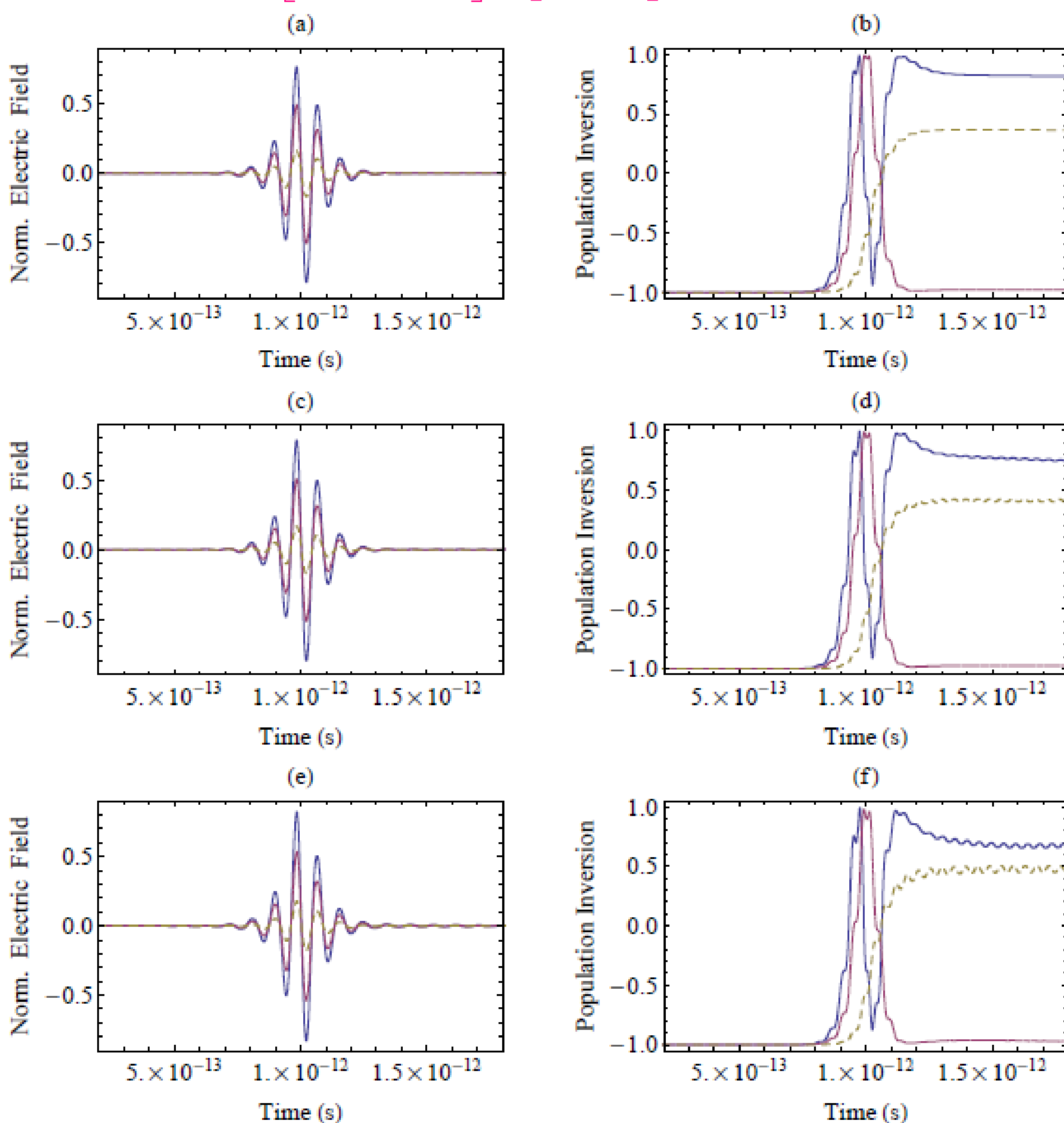


## BASIC RESULTS (Left):

In (a), (c) and (e) we present the normalized electric field for an initially  $2\pi$  pulse for  $t_p = 0.1$  ps. In (b), (d) and (f) we present the respective population inversion. In (a), (b)  $N = 10^9 \text{ cm}^{-2}$  in (c), (d)  $N = 2 \times 10^{11} \text{ cm}^{-2}$  and in (e), (f)  $N = 5 \times 10^{11} \text{ cm}^{-2}$ . The blue curves are for  $z = 20$  nm, the pink curves are for  $z = 320$  nm and the green curves are for  $z = 640$  nm.



**Upper figure:** Double symmetric semiconductor quantum well structure, energies and corresponding envelope functions. Lower subband (green line) and upper subband (red line) are shown.



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